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Sample Question Paper for Ph.D. (Mathematics)
SPSAT'18

INSTRUCTIONS

The test is 60 minutes long & consists of 40 multiple choice questions (MCQ) adding up to 40 marks.

1. Which form of electromagnetic radiation has the greatest energy?
   (a) Infrared radiation, approximately 10 nm (b) UV radiation, approximately 200 nm
   (c) Microwaves, approximately 10 mm (d) X-rays, approximately 100 pm

1. If $X=\{a,b\}, P(X)=\{\phi, X, \{a\}, \{b\}\}$, then which one of the following is a topology?
   (a) $T=\{\phi, \{a\}, \{b\}\}$ (b) $T=\{X, \{a\}, \{b\}\}$
   (c) $T=\{\phi, \{a\}\}$ (d) $T=\{\phi, X\}$

2. Which one of the following is not true?
   (a) a second countable space is always first countable
   (b) first countable space always implies second countable space
   (c) every subspace of a second countable space is second countable
   (d) a second countable space is always separable

3. Every metric space is not a
   (a) T_2- space (b) T_3- space (c) T_4- space (d) T_5- space

4. The product of finitely many compact spaces is
   (a) compact space (b) open set (c) null set (d) none of the above

5. Every closed interval in real line R is
   (a) compact space (b) hausdroff space (c) null set (d) none of the above

6. The minimizing curve must satisfy a differential equation, called as
   (a) lagrange's equation (b) euler-lagrange equation
   (c) gauss equation (d) none of the above

7. Rayleigh-Ritz method is used to
   (a) find maxima (b) find minima
   (c) solve boundary value problems (d) none of the above

8. The solution of integral equation $y(x)=1+\int_0^x y(t)\ dt$ is

(a) $y=e^x$ (b) $y=e^{-x}$ (c) $y=e^{x^2}$ (d) $y=e^{-x^2}$

9. Of all the parabolas which pass through (0, 0) & (1, 1), the one, which when rotated about the x – axis, generates a solid of revolution with least possible volume between x = 0 & x = 1 is:

(a) $y=5x^2-3x$ (b) $y=\frac{1}{2}(5x^2-3x)$

(c) $y=3x^2-5x$ (d) $y=\frac{1}{2}(3x^2-5x)$

10. The function $f(z)=|z|^2$ is differentiable at:

(a) $z=0$ (b) $z\neq 0$ (c) Nowhere (d) None of the above

11. The singular solution of the differential equation $x\left(\frac{dy}{dx}\right)^2-2y\left(\frac{dy}{dx}\right)+4x=0$

(a) $y=\pm x^2$ (b) $y=2x+3$

(c) $y=x(x-2)$ (d) $y=\pm 2x$

12. Two linearly independent solutions of Differential equation $(4D^2+4D+5)y=0$ are

(a) $e^{x/2}\cos x$ and $e^{x/2}\sin x$ (b) $e^{-x/2}\cos x$ and $e^{-x/2}\sin x$

(c) $e^{x/2}\cos x$ and $e^{-x/2}\sin x$ (d) $e^{-x/2}\cos x$ and $e^{x/2}\sin x$

13. The differential of all parabolas lying in a plane is

(a) $\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)^{-1/2}=0$ (b) $\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)^{-3/2}=0$

(c) $\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)=0$ (d) $\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)^{-1}=0$

14. The Differential equations of a family of orthogonal trajectories of the family of curves given by the equations $f\left(x,y,\frac{dy}{dx}\right)=0$ is

(a) $f\left(x, y, \frac{dx}{dy}\right) = 0$ (b) $f\left(x, y, \frac{1}{-\frac{dy}{dx}}\right) = 0$

(c) $f\left(x, y, \frac{x}{\frac{dx}{dy}}\right) = 0$ (d) $f\left(x, y, \frac{x}{-\frac{dy}{dx}}\right) = 0$

15. The one dimensional wave equation is given by

(a) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ (b) $\frac{\partial y}{\partial t} = a^2 \frac{\partial^2 y}{\partial x^2}$

(c) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial y}{\partial x}$ (d) $\frac{\partial y}{\partial t} = a^2 \frac{\partial y}{\partial x}$

16. Partial differential equation of the first order $P(x, y, z)z_x + Q(x, y, z)z_y = R(x, y, z)$ is

(a) Quasi linear (b) Non linear (c) Linear (d) None of the above

17. The Complementary function of the following partial differential equation

$$25r - 40s + 16t = 0$$

(a) $z = f_1(5y + 4x) + xf_2(5y + 4x)$ (b) $z = f_1(4y + 5x) + xf_2(4y + 5x)$

(c) $z = f_1(3y + 4x) + xf_2(3y + 4x)$ (d) $z = f_1(4y + 3x) + xf_2(4y + 3x)$

18. For the following partial differential equation

$\frac{\partial z}{\partial x} + 2xy^3 \frac{\partial z}{\partial y} = z^3$ General solution can be expressed in the form $F(u, v) = 0$ where

(a) $u(x, y, z) = x^2 + y^{-2}, \quad v(x, y, z) = x + z^{-2}$

(b) $u(x, y, z) = x^2 - y^2, \quad v(x, y, z) = x - z^{-2}$

(c) $u(x, y, z) = x^2 - \frac{y^2}{2}, \quad v(x, y, z) = x - \frac{z^{-2}}{2}$

(d) $u(x,y,z) = x^2 + \frac{y^{-2}}{2}, \quad v(x,y,z) = x + \frac{z^{-2}}{2}$

19. The particular integral of the following differential equation $\left(D^6 - D^4\right)y = x^2$ is:

(a) $-\left[\frac{x^6}{36} + \frac{x^4}{12}\right]$ (b) $-\left[\frac{x^6}{12} + \frac{x^4}{36}\right]$

(c) $-\left[\frac{x^6}{360} + \frac{x^4}{12}\right]$ (d) $-\left[\frac{x^6}{12} + \frac{x^4}{360}\right]$

20. The Lagrangian for simple harmonic oscillator which consist of mass m moving in a quadratic potential field, is $L(x,\dot{x}) = \frac{1}{2}m\,\dot{x}^2 - \frac{1}{2}kx^2$ then Hamiltonian is given by

(a) $H(x,p) = \frac{p^2}{2m} + \frac{1}{2}kx^2$ (b) $H(x,p) = \frac{p^2}{m} + \frac{1}{2}kx^2$

(c) $H(x,p) = \frac{p^2}{2m} + kx^2$ (d) $H(x,p) = \frac{p^2}{2m} + 2kx^2$

21. A box contains 40 numbered red balls & 60 numbered black balls. From the box, balls are drawn one by one at random without replacement till all balls are drawn. The probability that the last ball drawn is black equals

(a) 1/100 (b) 1/60 (c) 3/5 (d) 2/3

22. X & Y are independent random variables each having the density $f(t) = \frac{1}{\pi}\frac{1}{1+t^2}, \quad -\infty < t < \infty$

then the density function of $\frac{X+Y}{3}$ for $-\infty < t < \infty$ is given by

(a) $\frac{6}{p} \quad \frac{1}{4+9t^2}$ (b) $\frac{6}{p} \quad \frac{1}{9+4t^2}$

(c) $\frac{3}{p} \quad \frac{1}{1+9t^2}$ (d) $\frac{3}{p} \quad \frac{1}{9+t^2}$

23. Let $X \sim N(\mu, \Sigma)$ where $\mu = (1,1,1)$ & $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & c \\ 1 & c & 2 \end{pmatrix}$. The value of c such that $-X_2$ & $-X_1 + X_2 - X_3$ are independent is

(a) -2 (b) 0 (c) 2 (d) 1

24. Let $\{X_t\}$ & $\{Y_t\}$ be two independent pure birth processes with birth rates λ_1 & λ_2 respectively. Let $Z_t = X_t + Y_t$. Then

(a) $\{ Z_t \}$ is not a pure birth process

(b) $\{ Z_t \}$ is a pure birth process with birth rate $\lambda_1 + \lambda_2$

(c) $\{ Z_t \}$ is a pure birth process with birth rate min $(\lambda_1 . \lambda_2)$

(d) $\{ Z_t \}$ is a pure birth process with birth rate $\lambda_1 \lambda_2$

25. Let $X_1, \ldots, X_n$ be a random sample from N(θ, 1), where θ ∈ {1, 2}. Then which of the following statements about the maximum likelihood estimator (MLE) of θ is correct?

(a) MLE of θ does not exist (b) MLE of θ is $\bar{X}$

(c) MLE of θ exists but not $\bar{X}$ (d) MLE of θ is an unbiased estimator of θ

26. Customers arrive at an icecream parlour according to a Poisson process with rate 2. Service time distribution has density function $f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & x \le 0 \end{cases}$ upon being served a customer may rejoin the queue with probability 0.4. Independently of new arrivals; also a returning customer's service times same that of a new arriving customer. Customers behave independently of each other. Let X(t) = number of customers in the queue at time t. which among the following is correct?

(a) {X(t)} grows without bound with probability 1

(b) {X(t)} has stationary distribution given by $\pi_k = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^k, k = 0,1,2,.......$

(c) {X(t)} has stationary distribution given by $\pi_k = (0.1)(0.9)^k, k = 0,1,2,.......$

(d) {X(t)} has stationary distribution given by $\pi_k = (0.4)(0.6)^k, k = 0,1,2,.......$

27. Ten balls are put in 6 slots at random. Then the expected total number of balls in the two extreme slots is:

(a) 10/6 (b) 10/3 (c) 1/6 (d) 6/10

28. For a>0, the series $\sum_{n=1}^{\infty} a^{\ln n}$ is convergent if & only if

(a) $0 < a < e$ (b) $0 < a \le e$ (c) $0 < a < 1/e$ (d) $0 < a \le 1/e$

29. If $L = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n!}}$, then

(a) L =0 (b) L = 1 (c) 0 < L < ¥ (d) L = ¥

30. The sum of the series $\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+3}{3!} + ...$ equals

(a) e (b) e/2 (c) 3e/2 (d) 1 + e/2

31. If $f : G \to H$ be a group homomorphism. Then Ker f is a

(a) subgroup of G (b) subgroup of H

(c) normal subgroup of G (d) none of the above

32. Let (Z,+) be the additive group of integers & G be the multiplicative group of the fourth roots of unity. Let $f : Z \to G$ be a homomorphism given by f(x) = i^n Then ker f is

(a) empty set (b) [4m : m $\in$ Z]

(c) [2m+1 : m $\in$ Z] (d) none of the above

33. An example of a group homomorphism from the set of integers to itself is

(a) f(n) = -n for all n (b) f(n) = n+1 for all n

(c) f(n) = $(n+1)^2$ for all n (d) f(n) = n^2 for all n

34. Let G be a group of order 30. Let H & K be two normal subgroups of order 2 & 5 respectively. Then the order of the group $\frac{G}{HK}$ is

(a) 10 (b) 2 (c) 3 (d) 5

35. If a & a^2 are both generators of a cyclic group of order n, then

(a) n must be odd (b) n must be even

(c) n must be prime (d) n must not be a prime

36. In the symmetric group S_7, the order of the permutation (1 2 3 4) (5 6 7) is

(a) 1 (b) 2 (c) 5 (d) 12

37. If S & T are subsets of V(F), Then which one is wrong

(a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ (b) $L(S \cup T) = L(S) - L(T)$

(c) $S \text{ is a subspace of } V \Leftrightarrow L(S) = S$ (d) $L\{L(S)\} = L(S)$

38. In V(R), where R is the field of real numbers examines each of the following set of vectors & find which one is LI:

(a) {(1,2,0),(0,3,1),(-1,0,1)} (b) {(-1,2,1),(3,0,-1),(-5,4,3)}

(c) {(1,3,2),(1,-7,-8),(2,1,-1)} (d) {(1,2,1),(3,1,5),(3,-4,7)}

39. Which of the following statement is wrong?

(a) There exists a basis for each finite dimensional vector space

(b) If V(F) is a finite dimensional vector space, then any two bases of V have the same number of elements

(c) Every Linearly independent subset of a finitely generated vector space V(F) forms a part of basis of V

(d) Each set of (n+1) or more vectors of a finite dimensional vector space V(F) of dimension ‘n’ is LI

40. Which of the following statement is correct?

(i) The intersection of any two subspaces W_1 & W_2 of a vector space V(F) is also a subspace V(F)

(ii) The union of any two subspaces is a subspace iff one is contained in the other

(a) only (i) (b) both (i) & (ii) (c) only (ii) (d) none of the above